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A coordinate-dependent superspace deformation from string theory

León G. Aldrovandi, Fidel A. Schaposnik*and Guillermo A. Silva†

Departamento de Física, Facultad de Ciencias Exactas
Universidad Nacional de La Plata
Argentina
E-mail: aldrovandi@fisica.unlp.edu.ar, fidel@fisica.unlp.edu.ar, silva@fisica.unlp.edu.ar

Abstract: Starting from a type II superstring model defined on $R^{2,2} \times CY_6$ in a linear graviphoton background, we derive a coordinate dependent C-deformed $\mathcal{N} = 1$, $d = 2 + 2$ superspace. The chiral fermionic coordinates θ satisfy a Clifford algebra, while the other coordinate algebra remains unchanged. We find a linear relation between the graviphoton field strength and the deformation parameter. The null coordinate dependence of the graviphoton background allows to extend the results to all orders in α'.

Keywords: D-branes, Superstrings and Heterotic Strings, Superspaces.

*F.A.S. is associated with CICBA.
†G.A.S. is associated with CONICET

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1. Introduction

Turning on background fields in superstring models in the presence of D-branes leads to a deformation of the superspace geometry. In the case of a constant Neveu-Schwarz B-field background, the commutator of the space-time coordinates gets deformed by a central term (see [1] and references therein). If instead one considers a Ramond-Ramond background the odd coordinates of superspace become nonanticommuting [2]–[4] (for related work see also [12 – 14]).

In the latter case, if one considers a type II superstring in R^4 with $\mathcal{N} = 2$ supersymmetry in the presence of a constant self-dual graviphoton field, the effective fermionic coordinate algebra becomes

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta}. \tag{1.1}$$

The connection between the deformation symmetric matrix $C^{\alpha\beta}$ and the constant graviphoton field strength background $F^{\alpha\beta}$ is [2]–[5] and [15, 16]

$$C^{\alpha\beta} = {\alpha'}^2 F^{\alpha\beta}. \tag{1.2}$$

This relation is derived using the so-called hybrid formalism [6] and [7] which is particularly adapted to the case of superstrings in Ramond-Ramond background fields (see also [17] for related work).

The graviphoton field strength in (1.2) is selfdual and constant, therefore not contributing to the energy-momentum tensor and hence not backreacting on the flat Minkowski metric. Moreover, it does not affect the dilaton equations of motion so that such a background (flat metric + constant selfdual graviphoton) is an exact solution of the equations of motion to all orders in α'.

An interesting possibility is to consider more general graviphoton backgrounds which could, in principle, lead to a coordinate dependent deformation $C^{\alpha\beta}(y)$. In particular, motivated by a suggestion in [8], it was shown in [9] that, within the context of $\mathcal{N} = 1$ $d = 4$ euclidean supersymmetry, such a kind of deformation could be implemented for the

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Super Yang-Mills model provided the deformation parameters $C^{\alpha\beta}$ satisfy

$$\bar{\sigma}^{\mu}_{\dot{\alpha}\alpha} \frac{\partial C^{\alpha\beta}(y)}{\partial y^{\mu}}\bigg|_{\theta,\bar{\theta}} = 0 \tag{1.3}$$

with y^{μ} the standard hermitian (bosonic) chiral coordinates defined from the superspace coordinates $(x^{\mu}, \theta^{\alpha}, \bar{\theta}^{\dot{\alpha}})$ as

$$y^{\mu} = x^{\mu} + i\theta^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} \tag{1.4}$$

As shown in [9], only when condition (1.3) is satisfied the (antichiral) superfield strength $\bar{W}_{\alpha}$ transforms covariantly under supergauge transformations and then only in this case a deformed Super Yang-Mills theory can be consistently defined.

The question that we address in this paper is whether a coordinate dependent deformation such that

$$\{\theta^{\alpha}, \theta^{\beta}\} = C^{\alpha\beta}(y) \tag{1.5}$$

satisfying condition (1.3) can be obtained in superstring theory by considering a D-brane in a non-constant self-dual background $F^{\mu\nu}(y)$. As we shall see, by considering a type II superstring model defined on $R^{(2,2)}$ in a family of self-dual graviphoton backgrounds $F_{\mu\nu}(y)$, we find an affirmative answer to the questions (the choice of signature will be justified below).

The paper is organized as follows: in section 2 we present the worldsheet Lagrangian for a string coupled to a graviphoton background working in Berkovits hybrid formalism. We write down the lowest α' order equation for the graviphoton and, motivated by the pp-wave case, we choose a suitable class of solutions which allows us to integrate the equations of motion for the propagators. In section 3 we obtain the boundary conditions for open strings ending on a D3-brane filling the $R^{2,2}$ space-time and discuss the supersymmetry preserved. In section 4 we solve for the propagators and compute the superspace coordinate algebra. Section 5 contains a discussion of the results.

2. The worldsheet lagrangian

We shall consider a type II superstring model defined on $R^{(2,2)} \times CY_6$ whose $\mathcal{N} = 2$ supersymmetry in $d = 4$ dimensions is deformed by the presence of a self-dual graviphoton background. Since we are interested in Ramond-Ramond backgrounds, it will be convenient to work within Berkovits' hybrid formalism [6]–[7]. We start from the worldsheet Lagrangian

$$\mathcal{L}_0 = \frac{1}{\alpha'}\left(\frac{1}{2}\tilde{\partial}x^{\mu}\partial x_{\mu} + p_{\alpha}\tilde{\partial}\theta^{\alpha} + \bar{p}_{\dot{\alpha}}\tilde{\partial}\bar{\theta}^{\dot{\alpha}} + \tilde{p}_{\alpha}\partial\tilde{\theta}^{\alpha} + \tilde{\bar{p}}_{\dot{\alpha}}\partial\tilde{\bar{\theta}}^{\dot{\alpha}}\right) \tag{2.1}$$

here $\mu = 1,2,3,4$ and $\alpha, \dot{\alpha} = 1,2$ (for metric and spinor conventions see the appendix). The bar denotes space-time chirality while the tilde the worldsheet chirality. We parametrize the Euclidean signature worldsheet coordinates with z and $\tilde{z}$ and write $\partial = \partial/\partial z$ and $\tilde{\partial} = \partial/\partial\tilde{z}$. The canonical conjugates to the complex fermion variables $\theta, \tilde{\theta}, \bar{\theta}$ and $\tilde{\bar{\theta}}$ are denoted as $p, \tilde{p}, \bar{p}$ and $\tilde{\bar{p}}$. These conjugate momenta p's can be seen as the worldsheet versions of $-\partial_{\theta}|_x, -\partial_{\bar{\theta}}|_x, -\partial_{\tilde{\theta}}|_x$ and $-\partial_{\tilde{\bar{\theta}}}|_x$. We have not included in Lagrangian (2.1) neither

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the chiral boson nor compactification dependent terms since they will not be relevant for the following discussion.

Since we are interested in working in terms of chiral variables y^μ and derivatives with fixed y, we change variables to y^μ, q_α, $\tilde{q}_\alpha$, $\bar{d}_{\dot\alpha}$ and $\tilde{\bar{d}}_{\dot\alpha}$ given by

$$\begin{aligned}
y^\mu &= x^\mu + i\theta^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha} + i\tilde\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\tilde{\bar\theta}^{\dot\alpha} \\
\bar{d}_{\dot\alpha} &= \bar{p}_{\dot\alpha} - i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial x_\mu - \theta\theta\,\partial\bar\theta_{\dot\alpha} + \frac{1}{2}\bar\theta_{\dot\alpha}\partial(\theta\theta) \\
q_\alpha &= -p_\alpha - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial x_\mu + \frac{1}{2}\bar\theta\bar\theta\,\partial\theta_\alpha - \frac{3}{2}\partial(\theta_\alpha\bar\theta\bar\theta) \\
\tilde{\bar{d}}_{\dot\alpha} &= \tilde{\bar{p}}_{\dot\alpha} - i\tilde\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\tilde\partial x_\mu - \tilde\theta\tilde\theta\,\tilde\partial\tilde{\bar\theta}_{\dot\alpha} + \frac{1}{2}\tilde{\bar\theta}_{\dot\alpha}\tilde\partial(\tilde\theta\tilde\theta) \\
\tilde{q}_\alpha &= -\tilde{p}_\alpha - i\sigma^\mu_{\alpha\dot\alpha}\tilde{\bar\theta}^{\dot\alpha}\tilde\partial x_\mu + \frac{1}{2}\tilde{\bar\theta}\tilde{\bar\theta}\,\tilde\partial\tilde\theta_\alpha - \frac{3}{2}\tilde\partial(\tilde\theta_\alpha\tilde{\bar\theta}\tilde{\bar\theta})
\end{aligned} \tag{2.2}$$

so that Lagrangian (2.1) takes the form

$$\mathcal{L}_0 = \frac{1}{\alpha'}\left(\frac{1}{2}\tilde\partial y^\mu\partial y_\mu - q_\alpha\tilde\partial\theta^\alpha + \bar{d}_{\dot\alpha}\tilde\partial\bar\theta^{\dot\alpha} - \tilde{q}_\alpha\partial\tilde\theta^\alpha + \tilde{\bar{d}}_{\dot\alpha}\partial\tilde{\bar\theta}^{\dot\alpha}\right) \tag{2.3}$$

Here the $\bar{d}$'s and q's replace the p's and act as derivatives with respect to the fermionic coordinates but keeping y fixed (instead of fixed x as in the p's case). They are the worldsheet versions of the covariant derivative $\bar{D}$ and the supercharge Q in target space.

The coupling of the string to an arbitrary graviphoton field background is implemented by adding to the Lagrangian (2.3) the vertex operator for $F_{\mu\nu}$

$$V = \int d^2z\, q_\alpha\tilde{q}_\beta F^{\alpha\beta}(y) \tag{2.4}$$

where

$$F^{\alpha\beta}(y) = \frac{1}{2}\varepsilon^{\alpha\gamma}\left(\sigma^{\mu\nu}\right)_\gamma^{\ \beta} F_{\mu\nu}(y). \tag{2.5}$$

The total Lagrangian describing the coupling of the string to a graviphoton background is then

$$\mathcal{L} = \frac{1}{\alpha'}\left(\frac{1}{2}\tilde\partial y^\mu\partial y_\mu - q_\alpha\tilde\partial\theta^\alpha + \bar{d}_{\dot\alpha}\tilde\partial\bar\theta^{\dot\alpha} - \tilde{q}_\alpha\partial\tilde\theta^\alpha + \tilde{\bar{d}}_{\dot\alpha}\partial\tilde{\bar\theta}^{\dot\alpha} + \alpha' q_\alpha\tilde{q}_\beta F^{\alpha\beta}(y)\right). \tag{2.6}$$

As explained in the introduction, in order to get no back reaction on the flat metric, we only consider the coupling to a self-dual graviphoton background, this being only possible in $4+0$ and $2+2$ spaces.

Being Lagrangian (2.6) quadratic in q and $\tilde{q}$ one can easily integrate them out. Indeed, defining

$$Z[\theta,\tilde\theta;F(y)] \equiv \int D\tilde{q}Dq \exp\left(-\frac{1}{\alpha'}\int d^2z\left(-q_\alpha\tilde\partial\theta^\alpha - \tilde{q}_\alpha\partial\tilde\theta^\alpha + \alpha' q_\alpha\tilde{q}_\beta F^{\alpha\beta}(y)\right)\right) \tag{2.7}$$

one has

$$Z[\theta,\tilde\theta;F(y)] = \exp\left(-\int d^2z\frac{1}{\alpha'^2}F^{-1}_{\alpha\beta}(y)\partial\tilde\theta^\alpha\tilde\partial\theta^\beta\right). \tag{2.8}$$

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This leads to an effective Lagrangian of the form

$$\mathcal{L}_{eff} = \frac{1}{\alpha'}\left(\frac{1}{2}\tilde{\partial}y^{\mu}\partial y_{\mu} + \bar{d}_{\dot{\alpha}}\tilde{\partial}\bar{\theta}^{\dot{\alpha}} + \tilde{\bar{d}}_{\dot{\alpha}}\partial\tilde{\bar{\theta}}^{\dot{\alpha}} + \frac{1}{\alpha'}F^{-1}_{\alpha\beta}(y)\partial\tilde{\theta}^{\alpha}\tilde{\partial}\theta^{\beta}\right). \tag{2.9}$$

In a purely bosonic background, the lowest α' order equation of motion for the graviphoton field is just the sourceless Maxwell equation

$$\partial_{\mu}F^{\mu\nu} = 0\,. \tag{2.10}$$

This equation is trivially satisfied by any selfdual field strength which, as explained above, are precisely those which we consider here. The selfduality condition allows to write $F^{\mu\nu}$ in terms of a 2×2 symmetric matrix $F^{\alpha\beta}$ through the relation

$$F^{\mu\nu}(y) = \varepsilon_{\beta\gamma}(\sigma^{\mu\nu})_{\alpha}^{\ \gamma}F^{\alpha\beta}(y). \tag{2.11}$$

Being $F^{\mu\nu}$ real, the components of $F^{\alpha\beta}$ are also real in $2+2$ dimensions. It will be useful to rewrite (2.10) as

$$\bar{\sigma}^{\mu}_{\dot{\alpha}\alpha}\partial_{\mu}F^{\alpha\beta} = 0. \tag{2.12}$$

Introducing the null coordinates

$$\begin{aligned} U_1 &= y_1 + y_4,\ V_1 = y_1 - y_4 \\ U_2 &= y_2 + y_3,\ V_2 = y_2 - y_3, \end{aligned} \tag{2.13}$$

equations (2.12) for the graviphoton field read

$$\begin{aligned} \partial_{U_1}F^{11} - \partial_{V_2}F^{21} &= 0 \\ \partial_{U_2}F^{11} + \partial_{V_1}F^{21} &= 0 \\ \partial_{V_2}F^{22} - \partial_{U_1}F^{12} &= 0 \\ \partial_{V_1}F^{22} + \partial_{U_2}F^{12} &= 0. \end{aligned} \tag{2.14}$$

The relevant terms for our calculations, considering the Lagrangian (2.9) in terms of the light cone variables (2.13), are

$$\mathcal{L}^{chiral}_{eff} = -\frac{1}{4\alpha'}(\partial U_i\tilde{\partial}V_i + \tilde{\partial}U_i\partial V_i) + \frac{1}{\alpha'^2}F^{-1}_{\alpha\beta}(U,V)\,\partial\tilde{\theta}^{\alpha}\tilde{\partial}\theta^{\beta}. \tag{2.15}$$

The equations of motion for the bosonic (U,V) coordinates are

$$\begin{aligned} \partial\tilde{\partial}U_i &= -\frac{2}{\alpha'}\partial_{V_i}F^{-1}_{\alpha\beta}\,\partial\tilde{\theta}^{\alpha}\tilde{\partial}\theta^{\beta} \\ \partial\tilde{\partial}V_i &= -\frac{2}{\alpha'}\partial_{U_i}F^{-1}_{\alpha\beta}\,\partial\tilde{\theta}^{\alpha}\tilde{\partial}\theta^{\beta}\,. \end{aligned} \tag{2.16}$$

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We shall now choose a convenient graviphoton background satisfying the equations of motion (2.14). With the pp-wave case in mind [10], by taking F^{12} to be a constant one has from (2.14) that $F^{11} = F^{11}(V_1, V_2)$ and $F^{22} = F^{22}(U_1, U_2)$. Then, choosing also F^{11} to be constant we obtain from (2.16)

$$\Box U_i = 0. \tag{2.17}$$

Conformal invariance of the Lagrangian (2.9) (see the discussion section) allows one to choose the light cone gauge eliminating the oscillators dependence of only one of the null U_i coordinates, we choose this to be U_1 getting

$$U_1 = z + \bar{z}. \tag{2.18}$$

Taking $F^{22} = F(U_1)$ we have, in the light cone gauge, a gaussian Lagrangian. Let us also note that the choice of $F^{22} = F^{22}(U_1, U_2)$ and constant F^{11} makes only sense in $2+2$ signature since in the $4+0$ case F^{11} and F^{22} are related by complex conjugation. Finally, let us point that the choice (2.18) is consistent with the boundary conditions (3.2) (see below).

Summarizing, our graviphoton background takes the form

$$F^{\alpha\beta} = F_0^{\alpha\beta} + \delta_2^\alpha \delta_2^\beta F(U_1) \tag{2.19}$$

with $F_0^{\alpha\beta}$ a constant symmetric matrix and $F(U_1)$ an arbitrary function. The choice (2.19) plus conformal invariance results in a gaussian Lagrangian for the superspace coordinates in the light cone gauge.

3. D-brane boundary conditions

We shall take the worldsheet ending on a "D3-brane" whose worldvolume fills the $2+2$ space. The appropriate boundary conditions for the open string will be obtained by imposing the cancelation of the boundary terms in the equation of motion. When one maps the disc to the upper half plane the boundary conditions are imposed at $z = \tilde{z}$. From the Lagrangian (2.6) one gets the conditions

$$\begin{aligned}
&\int_{z=\tilde{z}} (d\tilde{z}\, \delta U_i\, \tilde{\partial} V_i - dz\, \delta U_i\, \partial V_i) = 0 \\
&\int_{z=\tilde{z}} (d\tilde{z}\, \delta V_i\, \tilde{\partial} U_i - dz\, \delta V_i\, \partial U_i) = 0 \\
&\int_{z=\tilde{z}} (dz\, q_\alpha \delta\theta^\alpha - d\tilde{z}\, \tilde{q}_\alpha \delta\tilde{\theta}^\alpha) = 0.
\end{aligned} \tag{3.1}$$

We impose Neumann boundary conditions for the bosonic coordinates of the "D3-brane"

$$(\partial - \tilde{\partial})U_i\Big|_{z=\tilde{z}} = 0, \qquad (\partial - \tilde{\partial})V_i\Big|_{z=\tilde{z}} = 0. \tag{3.2}$$

Concerning the fermionic sector, the cancelation of (3.1) can be achieved by demanding

$$(q_\alpha - \tilde{q}_\alpha)|_{z=\tilde{z}} = 0 \tag{3.3}$$

$$(\theta^\alpha - \tilde{\theta}^\alpha)\Big|_{z=\tilde{z}} = 0. \tag{3.4}$$

The equations of motion resulting from Lagrangian (2.6) take the form

$$\begin{aligned}\tilde{\partial}\theta^\alpha - \alpha' F^{\alpha\beta}\tilde{q}_\beta = 0\\ \partial\tilde{\theta}^\alpha + \alpha' F^{\alpha\beta} q_\beta = 0.\end{aligned} \tag{3.5}$$

Consistency with (3.3) demands that

$$(\tilde{\partial}\theta^\alpha + \partial\tilde{\theta}^\alpha)\Big|_{z=\bar{z}} = 0. \tag{3.6}$$

The boundary conditions (3.3)-(3.4) preserve half of the the supersymmetries. Indeed, in the absence of D-branes, Lagrangian (2.6) is invariant under the transformations

$$\delta\theta^\alpha = \epsilon^\alpha, \qquad \delta\tilde{\theta}^\alpha = \tilde{\epsilon}^\alpha \tag{3.7}$$

where ϵ^α and $\tilde{\epsilon}^\alpha$ are constants. These transformations give rise to conserved independent supercharges

$$Q_\alpha = \oint dz\, q_\alpha, \qquad \tilde{Q}_\alpha = \oint d\bar{z}\, \tilde{q}_\alpha. \tag{3.8}$$

When a D-brane is present the expression for the charges is

$$Q_\alpha = \int_C dz\, q_\alpha, \qquad \tilde{Q}_\alpha = \int_C d\bar{z}\, \tilde{q}_\alpha. \tag{3.9}$$

where C is a semi-circle centered on the origin. Conditions (3.3) imply that Q and $\tilde{Q}$ are equal and no longer independent.

4. Propagators and (anti)commutation relations

To obtain the anticommutation relations for the fermionic coordinates we need to compute the following propagators,

$$\begin{aligned}G_1^{\alpha\beta}(z,\tilde{z}|w,\tilde{w}) &\equiv \langle\theta^\alpha(z,\tilde{z})\theta^\beta(w,\tilde{w})\rangle\\ G_2^{\alpha\beta}(z,\tilde{z}|w,\tilde{w}) &\equiv \langle\theta^\alpha(z,\tilde{z})\tilde{\theta}^\beta(w,\tilde{w})\rangle\\ G_3^{\alpha\beta}(z,\tilde{z}|w,\tilde{w}) &\equiv \langle\tilde{\theta}^\alpha(z,\tilde{z})\tilde{\theta}^\beta(w,\tilde{w})\rangle\end{aligned} \tag{4.1}$$

which, according to Lagrangian (2.9), should obey the following differential equations

$$\partial_z\left(F^{-1}_{\alpha\beta}(z,\tilde{z})\tilde{\partial}_{\tilde{z}} G_i^{\beta\gamma}(z,\tilde{z}|w,\tilde{w})\right) = -\alpha'^2\delta_\alpha^\gamma\,\delta^2(z-w)\,\delta_i^2 \tag{4.2}$$

Conditions (3.4) and (3.6) at $w=\tilde{w}$ imply the following relations between the propagators G_i

$$G_1^{\alpha\beta}(z,\tilde{z}|w,\tilde{w}) = G_2^{\alpha\beta}(z,\tilde{z}|w,\tilde{w}) \tag{4.3}$$

$$G_2^{\alpha\beta}(w,\tilde{w}|z,\tilde{z}) = G_3^{\alpha\beta}(w,\tilde{w}|z,\tilde{z}) \tag{4.4}$$

$$\tilde{\partial}_{\tilde{w}} G_1^{\alpha\beta}(z,\tilde{z}|w,\tilde{w}) = -\partial_w G_2^{\alpha\beta}(z,\tilde{z}|w,\tilde{w}) \tag{4.5}$$

$$\tilde{\partial}_{\tilde{w}} G_2^{\alpha\beta}(w,\tilde{w}|z,\tilde{z}) = -\partial_w G_3^{\alpha\beta}(w,\tilde{w}|z,\tilde{z})\,. \tag{4.6}$$

The Grassmann character of θ and the conjugation relations among them (see the appendix) impose the constraints

$$
\begin{aligned}
G_1^{\alpha\beta}(z,\tilde z|w,\tilde w) &= -G_1^{\beta\alpha}(w,\tilde w|z,\tilde z) &(4.7)\\
G_3^{\alpha\beta}(z,\tilde z|w,\tilde w) &= -G_3^{\beta\alpha}(w,\tilde w|z,\tilde z) &(4.8)\\
(G_1^{\alpha\beta}(z,\tilde z|w,\tilde w))^* &= G_3^{\beta\alpha}(w,\tilde w|z,\tilde z) &(4.9)\\
(G_2^{\alpha\beta}(z,\tilde z|w,\tilde w))^* &= G_2^{\beta\alpha}(w,\tilde w|z,\tilde z) &(4.10)
\end{aligned}
$$

In order to go further, we shall consider the following graviphoton background

$$
F^{\alpha\beta}(U,V) = F_0^{\alpha\beta} + \lambda\, U_1\, \delta_2^\alpha \delta_2^\beta. \tag{4.11}
$$

Using the conformal invariance of the action one can fix $U_1 = z+\tilde z$ (more on the conformal invariance of the action in the discussion section). For this choice only the (22) component of the propagators computed in [2]-[5] for a constant background is modified. Moreover, since boundary conditions (3.4) and (3.6) are preserved under complex conjugation one needs only to solve for G_1 and G_2, this is because due to the complex conjugation properties in 2+2 G_3 can be obtained as the complex conjugate of G_1 (see eqn.(4.9)). The propagators satisfying the boundary conditions (3.4) and (3.6) are

$$
\begin{aligned}
G_1^{\alpha\beta}(z,\tilde z|w,\tilde w) &= -\frac{\alpha'^2}{2\pi}\left(F^{\alpha\beta}(z,w)\ln\frac{\tilde z - w}{z-\tilde w} + \lambda(\tilde z - z + w - \tilde w)\delta_2^\alpha\delta_2^\beta\right)\\
G_2^{\alpha\beta}(z,\tilde z|w,\tilde w) &= -\frac{\alpha'^2}{2\pi}\left(F^{\alpha\beta}(z,\tilde w)\ln\frac{|z - w|^2}{(z-\tilde w)^2} + \lambda(\tilde z - z + w - \tilde w)\delta_2^\alpha\delta_2^\beta\right)\\
G_3^{\alpha\beta}(z,\tilde z|w,\tilde w) &= -\frac{\alpha'^2}{2\pi}\left(F^{\alpha\beta}(\tilde z,\tilde w)\ln\frac{\tilde z - w}{z-\tilde w} + \lambda(\tilde z - z + w - \tilde w)\delta_2^\alpha\delta_2^\beta\right)
\end{aligned}
\tag{4.12}
$$

where

$$
F^{\alpha\beta}(z,w) = F_0^{\alpha\beta} + \lambda(z+w)\delta_2^\alpha\delta_2^\beta. \tag{4.13}
$$

Taking z and w to be on the boundary $z=\tilde z=\tau,\ w=\tilde w=\tau'$ one gets

$$
\langle\theta^\alpha(\tau)\theta^\beta(\tau')\rangle = \frac{i}{2}\alpha'^2 F^{\alpha\beta}(\tau,\tau')\,\mathrm{sgn}(\tau-\tau') \tag{4.14}
$$

from which one can get that the θ anticommutator as

$$
\begin{aligned}
\{\theta^\alpha,\theta^\beta\} &= \lim_{\epsilon\to 0}\Big(\theta^\alpha(\tau+\epsilon)\theta^\beta(\tau) + \theta^\beta(\tau)\theta^\alpha(\tau-\epsilon)\Big) &(4.15)\\
&= i\alpha'^2(F_0^{\alpha\beta} + \lambda U_1\delta_2^\alpha\delta_2^\beta) &(4.16)
\end{aligned}
$$

or

$$
\{\theta^\alpha,\theta^\beta\} = i\alpha'^2 F^{\alpha\beta}(y). \tag{4.17}
$$

Hence, as in the case of a constant background, the deformation of the θ anticommutator is proportional to the graviphoton field strength. From (2.10) one immediately verifies that the deformation parameter satisfies the condition (1.3) found in [9] by demanding consistency of the gauge theory. Although we have obtained this result for a field strength of the form (4.11), we expect that an analogous result should hold for more general cases. Also, since the $\bar\theta$ coordinates were not affected by the background coupling, the commutation relations of $\bar\theta$ with y^μ and θ are not modified. The same happens with the commutator between θ and y^μ

Concerning the commutation relations for bosonic space-time coordinates among themselves, the relevant propagators to consider are

$$\begin{aligned} K_1^{ij}(z,\tilde z|w,\tilde w) &= \langle U_i(z,\tilde z)U_j(w,\tilde w)\rangle \\ K_2^{ij}(z,\tilde z|w,\tilde w) &= \langle U_i(z,\tilde z)V_j(w,\tilde w)\rangle \\ K_3^{ij}(z,\tilde z|w,\tilde w) &= \langle V_i(z,\tilde z)V_j(w,\tilde w)\rangle \end{aligned} \tag{4.18}$$

which, according to Lagrangian (2.9) should obey the following equations

$$\begin{aligned} \Box_z K_1^{ij}(z,\tilde z|w,\tilde w) &= 0 \\ \Box_z K_2^{ij}(z,\tilde z|w,\tilde w) &= 2\alpha'\delta^{ij}\delta^2(z-w) \\ \Box_w K_2^{ij}(z,\tilde z|w,\tilde w) &= 2\alpha'\delta^{ij}\delta^2(z-w) - \\ &\qquad \frac{2}{\alpha'}\langle U_i(z,\tilde z)\,\partial_{U_j}F^{-1}_{\alpha\beta}(w,\tilde w)\,\partial\tilde\theta^\alpha(w,\tilde w)\tilde\partial\theta^\beta(w,\tilde w)\rangle \\ \Box_z K_3^{ij}(z,\tilde z|w,\tilde w) &= \frac{2}{\alpha'}\langle \partial_{U_i}F^{-1}_{\alpha\beta}(z,\tilde z)\tilde\partial\theta^\alpha(z,\tilde z)\partial\tilde\theta^\beta(z,\tilde z)\,V_j(w,\tilde w)\rangle \end{aligned} \tag{4.19}$$

here $\Box_z = \partial_z\tilde\partial_{\tilde z}$. Divergent $\delta(0)$ terms arise in (4.19) from tadpole graph when contracting derivatives of θ fields in the last two lines. They are put to zero by an appropriate regularization. Once this is done, equations (4.19) are replaced by

$$\begin{aligned} \partial_z\tilde\partial_{\tilde z}K_1^{ij}(z,\tilde z|w,\tilde w) &= 0 \\ \partial_z\tilde\partial_{\tilde z}K_2^{ij}(z,\tilde z|w,\tilde w) &= 2\alpha'\delta^{ij}\delta^2(z-w) \\ \partial_z\tilde\partial_{\tilde z}K_2^{ij}(w,\tilde w|z,\tilde z) &= 2\alpha'\delta^{ij}\delta^2(z-w) \\ \partial_z\tilde\partial_{\tilde z}K_3^{ij}(z,\tilde z|w,\tilde w) &= 0 \end{aligned} \tag{4.20}$$

which are just the equations one obtains in the constant $F_{\mu\nu}$ case leading to trivial commutation relations for the chiral coordinates y^μ [4], so that also in our coordinate dependent background one has

$$[y^\mu, y^\nu] = 0 \tag{4.21}$$

5. Discussion

Starting from a type II superstring model defined on $R^{2,2}\times CY_6$ we have constructed a deformed $\mathcal{N}=1$, $d=2{+}2$ superspace with a coordinate dependent deformation. Indeed, by turning on a self dual linear graviphoton background and using Berkovits hybrid formalism we have been able to compute the propagators for the coordinates of superspace and from them infer the deformed algebra of the supercoodinates.

The resulting deformation is of the same type as that proposed in [9]: the chiral fermionic coordinates θ are not Grassman variables but satisfy a Clifford algebra of the type $\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta}(y)$ while the other coordinates, y, $\bar{\theta}$, remain (anti)commuting. We find that a linear relation between the deformation parameter and the graviphoton field strength $C^{\alpha\beta}(y) = \alpha'^2 F^{\alpha\beta}(y)$ holds, as in the case of a constant background [2]-[4]. This relation implies that the deformation parameter satisfies the condition $\partial_\mu C^{\mu\nu} = 0$, also required in order to define a consistent Super Yang-Mills theory in such a deformed superspace [9].

The results above were obtained to the lowest α' order. In order to extend its validity to all orders, one should address the problem of conformal invariance of the theory defined by Lagrangian (2.6). Now, one can easily see that conformal invariance is guaranteed to all orders. Indeed, our choice of background is independent of the null coordinates V_1, V_2 (or, alternatively, U_1, U_2). Then, when one splits the U_i fields in the form $U_i = U_i^b + U_i^q$, with U_i^b a classical background and U_i^q the quantum fluctuations, this last has nothing to contract with and hence it does not contribute to the effective action. Then, after setting $U_i = U_i^b$, one ends with a Lagrangian (2.6) quadratic in the quantum fields. It is then easy to check that this quadratic Lagrangian has no conformal anomaly[1].

Concerning the deformed superalgebra we obtained, the following comments are worth mentioning. First, note this algebra for the supercoordinates implies that the ordinary bosonic coordinates $x = y - i\theta\sigma\bar{\theta}$ are noncommutatives and its deformation parameter is also coordinate dependent. Also, it is interesting to note that the resulting deformation can be easily generalized. Indeed, as shown in [2], a nonvanishing commutator between the bosonic chiral coordinates y can be obtained by turning on a NS-NS two form B while a similar non trivial result is gotten for the $y-\theta$ commutator through the gravitini Ψ. Finally, note that the linear dependence of the C parameter with the space-time coordinate is very similar to an old proposal of Schwarz and van Nieuwenhuizen [11] where they analyze a possible substructure of the space-time through the relation $\{\theta^\alpha, \theta^\beta\} = \gamma_\mu^{\alpha\beta} x^\mu$.

The study of a coordinate dependent C-deformation, started in [9] and continued in the present work was prompted by the observation in [8] on the connection between this deformation and the spectral degeneracy in SUSY gluodynamics. Now that we have obtained such deformation starting from a superstring theory, this connection should be more thoroughly studied. We hope to report on this issue in a forthcoming work.

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A. Spinors in 2+2

In $d = 2+2$ a Majorana-Weyl (real and chiral matrix being diagonal) representation for the gammas is

$$\Gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \sigma^\mu = \left(1, i\sigma^2, \sigma^1, \sigma^3\right), \quad \bar{\sigma}^\mu = \left(-1, i\sigma^2, \sigma^1, \sigma^3\right) \tag{A.1}$$

where $\mu, \nu = 1, 2, 3, 4$ and the metric is $\eta = (- - ++)$[2].

[1]We thank N. Berkovits for explaining us this point.

[2]Note that in $2+2$ space all σ^μ are real, so $(\sigma^\mu)^* = \sigma^\mu$ (cf. with the Lorentzian case where the σ^μ are Hermitic, this is $(\sigma^\mu)^* = (\sigma^\mu)^T$). No simple relation exist in Euclidean space.

It then follows that

$$C_{ab} = \begin{pmatrix} -\epsilon^{\alpha\beta} & 0 \\ 0 & -\epsilon_{\dot{\alpha}\dot{\beta}} \end{pmatrix}, \quad \epsilon^{\alpha\beta} = \epsilon_{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = i\sigma^2 \tag{A.2}$$

The inverse spinor metrics are defined as

$$\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \delta_{\alpha}^{\gamma} \Longrightarrow \epsilon_{\alpha\beta} = -\epsilon^{\alpha\beta} \tag{A.3}$$

$$\epsilon_{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\beta}\dot{\gamma}} = \delta_{\dot{\alpha}}^{\dot{\gamma}} \Longrightarrow \epsilon_{\dot{\alpha}\dot{\beta}} = -\epsilon^{\dot{\alpha}\dot{\beta}} \tag{A.4}$$

which we use to define

$$C^{ab} = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot{\alpha}\dot{\beta}} \end{pmatrix}, \quad \epsilon_{\alpha\beta} = \epsilon^{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -i\sigma^2 \tag{A.5}$$

Raising and lowering of indices are defined as

$$\psi^{\alpha} = \epsilon^{\alpha\beta}\psi_{\beta}, \quad \psi_{\alpha} = \epsilon_{\alpha\beta}\psi^{\beta} \tag{A.6}$$

The index structure for σ^{μ}, $\bar{\sigma}^{\mu}$ is

$$\sigma^{\mu}{}_{\alpha\dot{\alpha}}, \qquad \bar{\sigma}^{\mu\,\dot{\alpha}\alpha} \tag{A.7}$$

and moreover they are related by rising/lowering indices as

$$\sigma^{\mu\ \alpha\dot{\alpha}} = \epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}}\sigma^{\mu}{}_{\beta\dot{\beta}} = \bar{\sigma}^{\mu\,\dot{\alpha}\alpha}\,. \tag{A.8}$$

Since in $2+2$ the matrices $\sigma^{\mu\nu} = \frac{1}{4}(\sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu})$ are real, we conclude that χ_{α} and $(\chi_{\alpha})^{*}$ transform in the same way under $Spin(2,2)$. This can be seen as the reason for the existence of Majorana-Weyl spinors in 2+2. It is then consistent to impose the condition $(\chi_{\alpha})^{*} = \chi_{\alpha}$. Complex conjugation in $2+2$ does not change chirality (as in $3+1$) neither it changes the index position (as in $4+0$). When acting on a product of spinors we define it to invert their order

$$(\theta\chi)^{*} = (\theta^{\alpha}\chi_{\alpha})^{*} = (\chi_{\alpha})^{*}(\theta^{\alpha})^{*} = \chi_{\alpha}\theta^{\alpha} = -\theta^{\alpha}\chi_{\alpha} = -\theta\chi \tag{A.9}$$

$$(\theta\sigma^{\mu}\bar{\chi})^{*} = (\theta^{\alpha}\sigma^{\mu}{}_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}})^{*} = (\bar{\chi}^{\dot{\alpha}})^{*}(\theta^{\alpha})^{*}\sigma^{\mu}{}_{\alpha\dot{\alpha}} = -\theta\sigma^{\mu}\bar{\chi} \tag{A.10}$$

in these relations we have taken the spinors to be MW and used that the σ^{μ} matrices are real. Although working with real spinors, it is $i\theta\chi$ and $i\theta\sigma^{\mu}\bar{\chi}$ that are real.

The conjugation properties of the variables used in the text, which correspond to a pair of MW spinors or just simply one complex Weyl spinor, are

$$(\theta_{\alpha})^{*} = \tilde{\theta}_{\alpha}, \qquad (p_{\alpha})^{*} = -\tilde{p}_{\alpha} \tag{A.11}$$

Similar relations hold for dotted indices.

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